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Prediction of Transonic Flutter for a Supercritical Wing by Modified Strip Analysis

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The experiments of Farmer, Hanson, and Wynne showed that use of a supercritical airfoil can adversely affect wing flutter speeds in the transonic range. Inasmuch as adequate theories for three-dimensional unsteady transonic flow are not yet available, the modified strip analysis published by Yates in 1958 has been used to predict the transonic flutter boundary for the supercritical wing tested by Farmer, Hanson, and Wynne. The steady-state spanwise distributions of the section lift-curve slope and aerodynamic center, required as input for the flutter calculations, were obtained from pressure distributions measured by Harris. The calculated flutter boundary is in excellent agreement with experiment in the subsonic range. In the transonic range, a "transonic bucket" is calculated which closely resembles the experimental one with regard to both shape and depth; however, it occurs at about 0.04 Mach number lower than that found experimentally.

Nomenclature

$a_{c,n}$ = nondimensional distance from midchord to section aerodynamic center measured perpendicular to elastic axis, positive rearward, fraction of semichord b

b_r = semichord of wing measured perpendicular to elastic axis at spanwise reference station $\eta=0.75$ ($b_r=0.14948$ m)

$C_{l_{\alpha,n}}$ = section lift-curve slope for a section perpendicular to elastic axis

C_p = pressure coefficient

H = translational displacement of wing at elastic axis, positive downward, fraction of reference semichord b_r

k_{nr} = reduced frequency based on spanwise reference station ($\eta=0.75$) and on velocity component normal to elastic axis, $b_r\omega/V\cos\Lambda_{ea}$

M = freestream Mach number

m_r = mass of wing per unit span at spanwise reference station ($\eta=0.75$) ($m_r=2.57$ kg/m)

q = freestream dynamic pressure

V = freestream speed

X = streamwise distance from local leading edge, fraction of local chord

Y = spanwise station, fraction of semispan

α = angle of attack

η = nondimensional coordinate measured from wing root along elastic axis, fraction of elastic axis length

θ = torsional displacement of wing about elastic axis, positive leading edge up

Λ_{ea} = sweep angle of elastic axis ($\Lambda_{ea}=40.03$ deg)

μ_r = mass ratio based on spanwise reference station ($\eta=0.75$), $m_r/\pi\rho b_r^2$

ρ = freestream density

ω = circular frequency of vibration

ω_r = reference frequency, frequency of first uncoupled torsional mode of wing ($\omega_r=227.23$ rad/s)

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Introduction

THE experiments of Farmer, Hanson, and Wynne,[1] which determined the flutter characteristics of a supercritical wing and a dynamically similar conventional wing having the same planform, showed that use of a supercritical airfoil can adversely affect wing flutter speeds in the transonic range.

Although it was evident from aerodynamic data available more than 10 years ago[2] that this adverse effect should occur, the results of Ref. 1 aroused considerable concern, which in turn led to renewed interest in predicting the flutter characteristics of supercritical wings. Inasmuch as conventional lifting-surface theories do not lead to accurately predicted transonic flutter characteristics, and because adequate theories for three-dimensional unsteady transonic flow have not yet been developed, the modified strip analysis (first published by Yates[3] in 1958) has been used here to predict the transonic flutter boundary for the supercritical wing (Fig. 1) which was investigated in Ref. 1. McGrew et al.[4] have also calculated flutter characteristics for this wing but by use of subsonic doublet-lattice aerodynamics in combination with an extensive set of "weighting factors."

The modified strip analysis used here has given good flutter results for a broad range of swept and unswept wings at speeds up to hypersonic,[5] including effects of wing thickness[6,7] and angle of attack.[8] In particular, it was used successfully to calculate transonic flutter characteristics for some swept wings with conventional airfoils.[9]

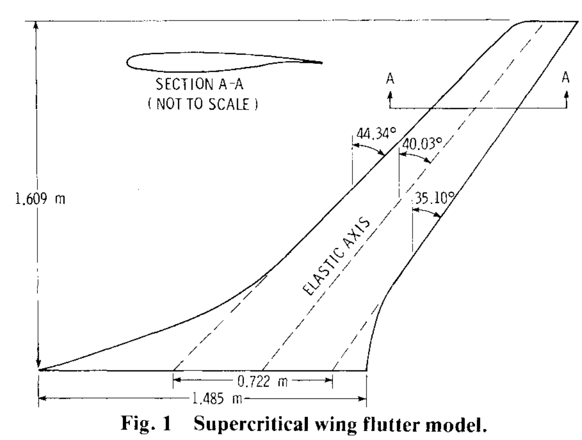

Fig. 1 Supercritical wing flutter model.

In addition to results calculated for comparison with the experimental data of Ref. 1, results are also shown herein that more broadly define the flutter characteristics of this wing in terms of the variations of flutter parameters with mass ratio at constant Mach number and with Mach number at constant mass ratio.

Flutter Model Parameters

The geometric, elastic, and inertial properties used in the present calculations are measured values for the supercritical wing flutter model of Ref. 1. The model geometry is shown in Fig. 1. The frequencies and node lines of the first six measured natural vibration modes and the modal deflections are presented, respectively, in Figs. 1b and 2 of Ref. 10. The corresponding generalized masses were determined by the method of displaced frequencies.[11]

These six measured modes were used in all of the flutter calculations shown herein. Some collateral flutter calculations, made with up to 12 vibration modes calculated with the NASTRAN¶ finite-element structural analysis, indicated that 6 modes were sufficient to converge flutter speeds and frequencies within 2%. Flutter calculations with the measured modes, however, appeared to converge more rapidly than those with the NASTRAN modes. Consequently, the six-measured-mode results presented here are considered to converge within about 1%.

Flutter Analysis

Method

The modified strip analysis[3] used here is formulated for wing strips oriented normal to the elastic axis and is based on stripwise application of Theodorsen-type aerodynamics[12] in which the lift-curve slope of 2π and aerodynamic center at the quarter chord are replaced, respectively, by the lift-curve slope and the aerodynamic center for the same strip of the three-dimensional wing at the appropriate Mach number. The downwash collocation point, where the downwash induced by the aerodynamic load is set equal to the kinematic downwash, is modified accordingly. The circulation function is modified for compressibility by use of two-dimensional unsteady compressible-flow theory.[13] Further description and discussion of this method are contained in Refs. 3, 5, 9, and 14.

Aerodynamic Parameters

The required spanwise distributions of the section lift-curve slope and the aerodynamic center were obtained from steady-state surface pressure measurements made by Harris[2] in the Langley 8-Foot Transonic Pressure Tunnel and in the Langley 16-Foot Transonic Tunnel. Upper- and lower-surface pressures were measured along streamwise wing sections at six stations along the semispan. Typical pressure distributions are shown in Fig. 2 for $M = 0.99$. The complete set of pressure distributions used for Mach numbers 0.25-1.20 are included in Ref. 10. Since the flutter tests were conducted at angles of attack near zero, the pressure data for the two angles of attack nearest zero were selected for use in the flutter calculations. The boundary-layer transition strips on the pressure model were also represented on the flutter model.

Values of the section lift-curve slope and section aerodynamic center for 40 wing sections equispaced along the elastic axis and oriented perpendicular to the elastic axis were required as input for the flutter calculations. These values were obtained by differencing the corresponding values of the section lift coefficient and pitching-moment coefficient for the two angles of attack shown in Fig. 2 and on each part of Figs. 3 and 4 of Ref. 10. The section lift and moment coefficients were calculated by 8 point Gaussian integration of the lifting pressure along each of the 40 wing sections.

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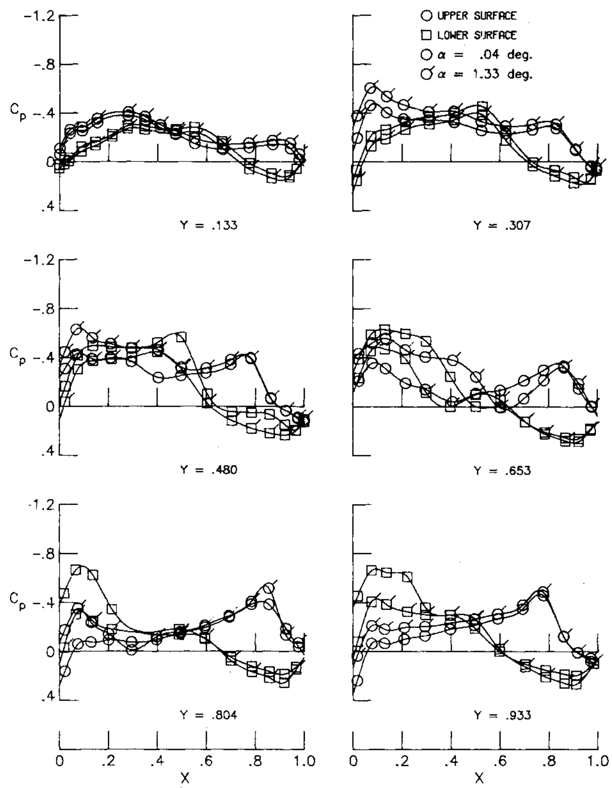

Fig. 2 Pressure distribution measured in 8-Foot Transonic Pressure Tunnel ($M = 0.99$, $q = 28.7$ kPa).

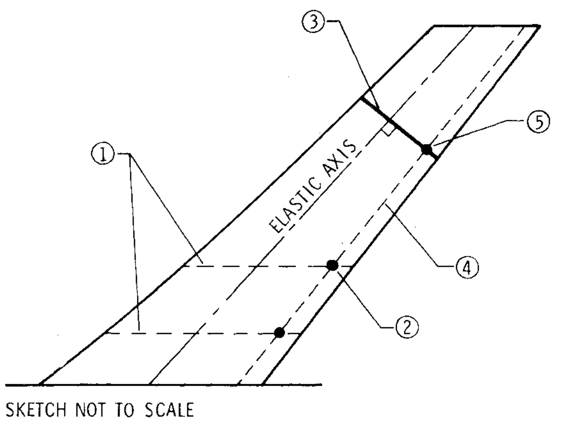

Fig. 3 Procedure for obtaining values of lifting pressure: 1) two of six streamwise lines along which pressures were measured, 2) one of eight points on each streamwise line at which pressures were interpolated, 3) one of 40 wing sections on which lift and moment coefficients were required, 4) one of eight lines of constant quadrature abscissa, and 5) one of eight quadrature points on each of 40 wing sections.

The procedure for obtaining values of lifting pressure at the 8 quadrature abscissas on each of the 40 wing sections from the measured pressures is described in the following discussion and Fig. 3. Since measured pressures were available only along six streamwise lines (see Fig. 3), it was necessary to interpolate the upper- and lower-surface pressures streamwise (Fig. 2) for the eight Gaussian-quadrature abscissas using the spline curves shown in these figures. For each of the quadrature abscissas, the resulting 6 values of lifting pressure (lower-minus upper-surface pressure) were spline-interpolated from root to tip, and values were thus obtained at the required 40 wing sections.

The distributions of section lift-curve slope and section aerodynamic center thus obtained are shown in Figs. 4 and 5. The change in the slopes of the curves inboard is caused by use of the dashed planform in Fig. 1 as reference while preserving

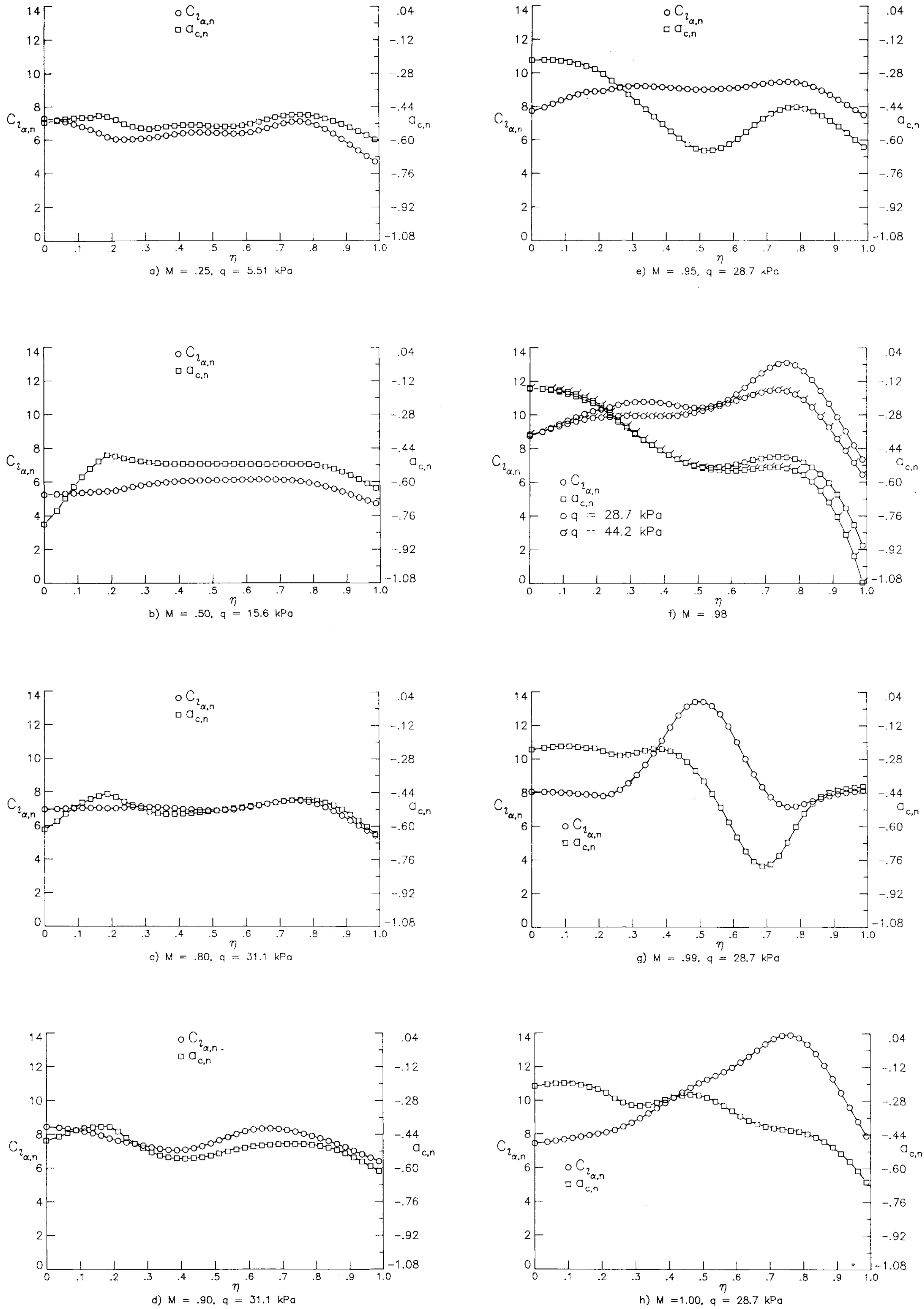

Fig. 4 Aerodynamic parameters for flutter analysis from 8-ft tunnel data.

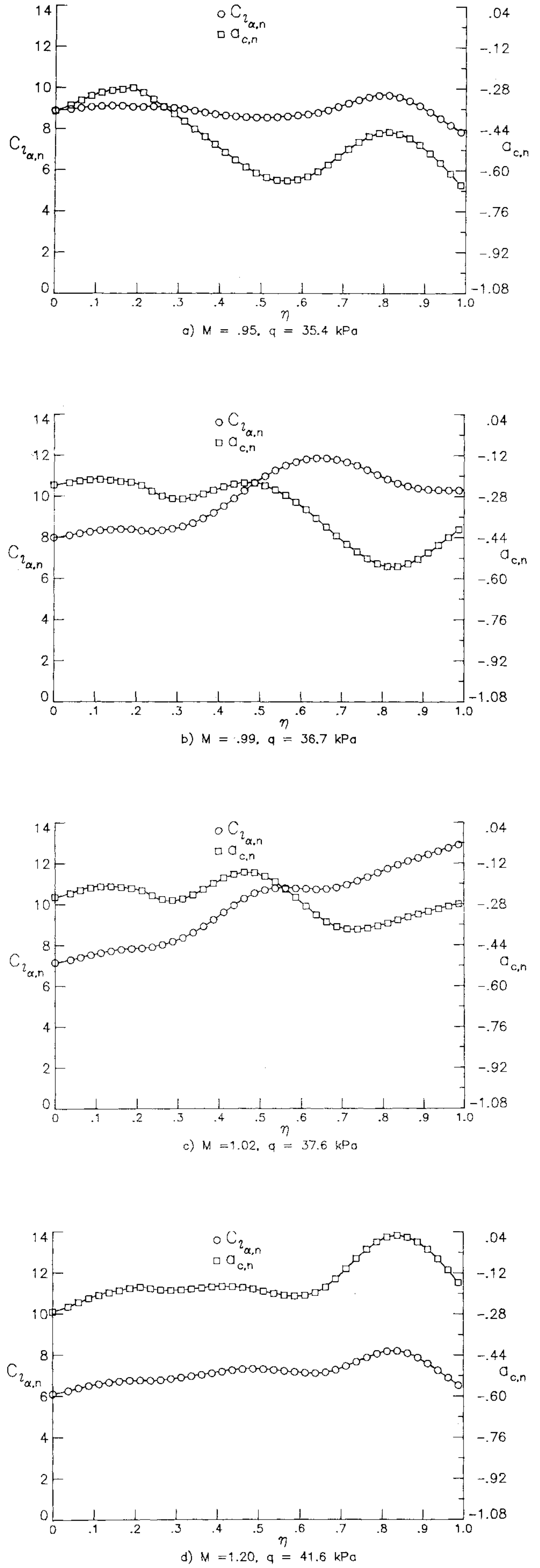

Fig. 5 Aerodynamic parameters for flutter analysis from 16-ft tunnel data.

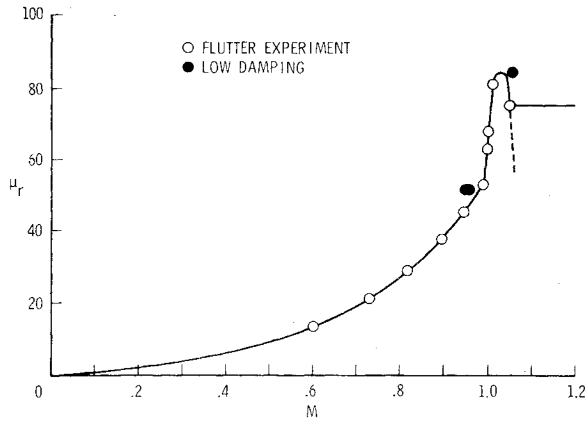

Fig. 6 Mass ratios for flutter experiments.

the section lift and moment values for the wing with glove. The spline-curve extrapolations beyond the aft-most experimental points in Fig. 2 generally indicate nonvanishing lifting pressures at the wing trailing edge. To examine the extent to which the calculated flutter characteristics were affected by the pressure-data fairing near the trailing edge, some collateral calculations were made with a Kutta condition imposed to force the lifting pressure to zero at the trailing edge. The resulting changes in calculated flutter characteristics were negligible.

Pressure data[2] from the tests in the 8-Foot Transonic Pressure Tunnel for Mach numbers up to 1.0 are available for two levels of dynamic pressure at each Mach number. Consequently, it is possible to assess the effect on aerodynamic parameters (and hence on flutter) of static aeroelastic deformation of the pressure model, together with an accompanying change in Reynolds number.** The comparison in Fig. 4f is typical. Increasing the dynamic pressure reduces the lift-curve slope and moves the aerodynamic center forward, especially over the outboard portions of the wing. These changes have opposing effects on flutter speed, and the resulting changes in calculated flutter characteristics are quite minor. Nevertheless, the lower value of dynamic pressure was selected for all of the flutter calculations presented herein because those pressure data should be less affected by deformation than those obtained at the higher dynamic pressure. The effect of the associated change in Reynolds-number is considered to be quite small since the previously indicated changes in aerodynamic parameters caused by changing dynamic pressure are in a direction opposite to that anticipated from Reynolds-number change alone.

Mass Ratio

The mass ratios used in flutter calculations made for direct comparison with the experimental flutter data of Ref. 1 were taken from a curve faired through the experimental mass ratios (Fig. 6). Inasmuch as no flutter data were obtained at Mach numbers higher than 1.055, the mass ratio for that Mach number was used for all higher Mach numbers. In addition, the mass ratio was varied parametrically up to a value of 98 for each Mach number in order to trace out "slices" of a flutter surface (flutter-speed index as a function of Mach number and mass ratio) such as that discussed in Appendix C of Ref. 6.

Results and Discussion

Values of flutter-speed index and flutter-frequency ratio calculated with mass ratios taken from the curve in Fig. 6 are

**Reynolds numbers of 2.2×10^6 and 3.3×10^6 at $M = 0.90$ (based on mean geometric chord) are representative for the 8-ft tunnel tests. The corresponding Reynolds numbers for the flutter tests were near the higher value.

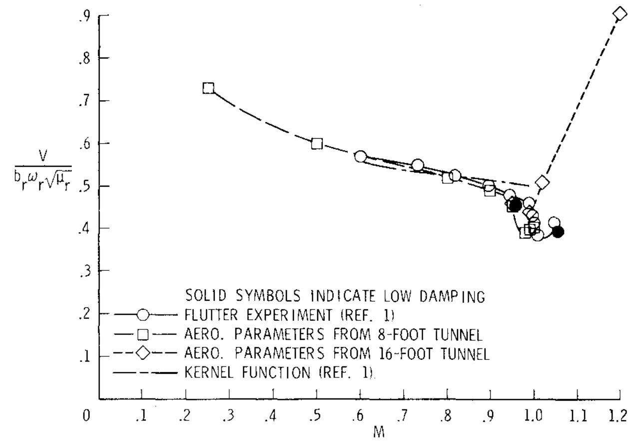

Fig. 7 Comparison of calculated and measured flutter speeds.

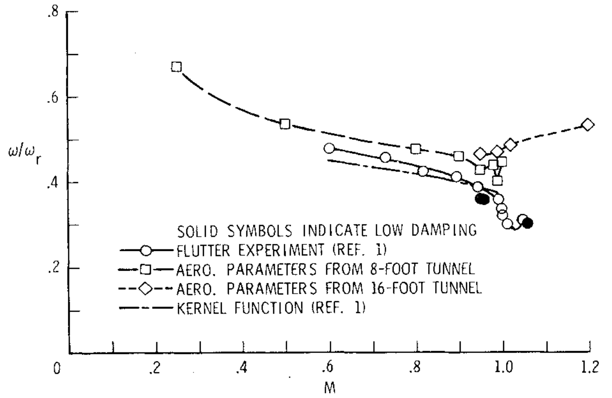

Fig. 8 Comparison of calculated and measured flutter frequencies.

compared with the experimental data[1] in Figs. 7 and 8, respectively. Agreement between calculated and measured flutter boundaries (Fig. 7) is excellent in the subsonic range. In the transonic range, a "transonic bucket" is calculated which closely resembles the experimental one with regard to both shape and depth. However, the calculated bucket occurs at about 0.04 Mach number lower than the experimental one. The reason for this difference is not known with certainty. There is some indication, however, that the difference may be associated with the flutter-model scale effect in the wind tunnel. A limited number of flutter points were measured with a second model of the wing with conventional airfoil[1] but 0.4 times as large as that reported in Ref. 1 (and 0.4 times as large as the supercritical wing model). The data for the smaller model showed the bottom of the transonic bucket to be shifted down about 0.04 in Mach number relative to that for the larger model. It is not likely that this difference was caused by the difference in blockage because blockage would be expected to cause the larger model to encounter transonic effects at lower Mach numbers than the smaller model. It is noted that, relative to tunnel dimensions, the pressure model is intermediate in size between the two conventional-airfoil flutter models and smaller than the supercritical wing model.

It is also possible that shock motion may contribute to the slightly early appearance of the calculated transonic bucket. Use of aerodynamic parameters obtained from measured steady-state pressure distributions in the present flutter calculations implies that both shock motions and viscous effects are incorporated. Moreover, these effects are not represented on a purely quasi-steady basis but are implicitly combined with other aerodynamic effects and subjected to the same attenuations and phase shifts. It is well known (e.g., Ref. 15) that the variation of shock motion and associated phase lag with frequency may differ appreciably from the corresponding attenuations and lags for unsteady shock-free

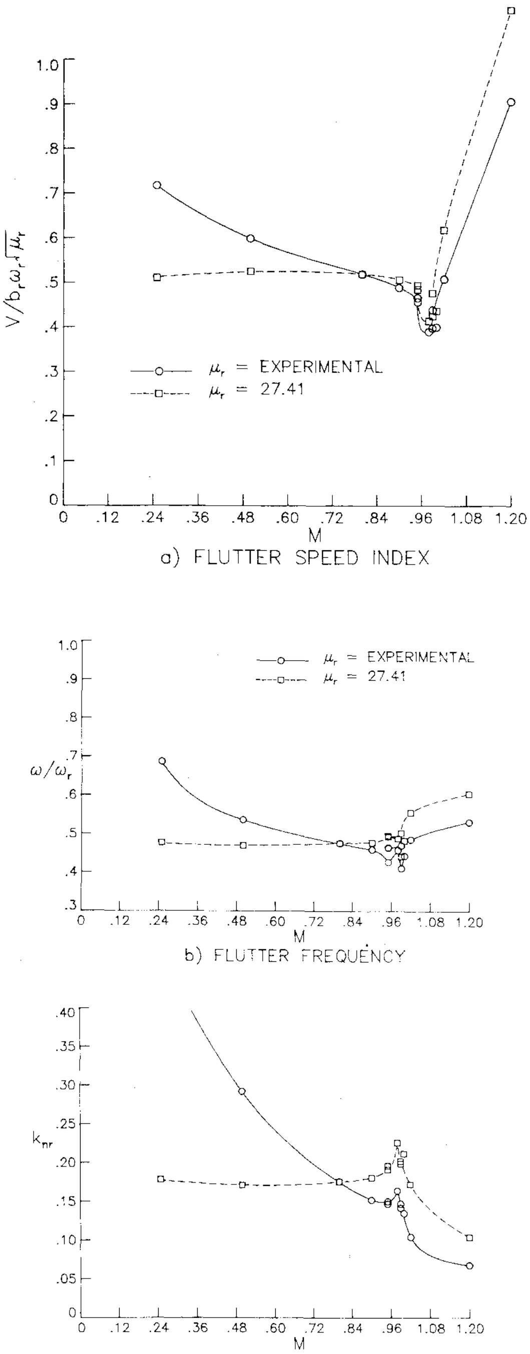

Fig. 9 Comparison of calculated flutter characteristics for constant and varying mass ratio.

flow. These differences, however, should be of only minor importance in the present calculations. Shock waves are relatively weak throughout the scope of this investigation because angles of attack are small and because the subject wing was designed to have a high drag-rise Mach number.

The calculated flutter frequencies (Fig. 8) are slightly higher than experimental values up to Mach number 0.95, as was the case in Ref. 4. The reason for the somewhat erratic variation between Mach numbers 0.95 and 1.00 (square symbols in Fig. 8) is not clear. In that Mach number range, however, aerodynamic data are available from both the 8-ft and 16-ft tunnels. Although the calculated flutter speeds are in good agreement with each other at the two duplicated Mach numbers ($M = 0.95$ and 0.99) (Fig. 7), the flutter frequencies obtained from the 16-ft tunnel data (diamond symbols) are higher and farther from flutter experimental data than those calculated with 8-ft tunnel data (square symbols). The cause of this difference is entirely aerodynamic, which may indicate that there is also a model scale effect on flutter frequency. Relative to tunnel size, the pressure model in the 16-ft tunnel was smallest; the same pressure model in the 8-ft tunnel was intermediate; and the flutter model in the Transonic Dynamics Tunnel was largest. Thus, if extrapolation of flutter frequency with respect to model size is valid, lower calculated flutter frequencies would be expected if the pressure model were the same size as the flutter model relative to the tunnel dimensions. This is, of course, speculative and empirical since no physical mechanism to produce this effect is postulated.

The upturn in the flutter speed and flutter frequency (Figs. 7 and 8) as Mach number decreases to 0.25 is caused by the accompanying decrease in mass ratio. This effect is illustrated more clearly in Fig. 9 where the calculated flutter characteristics from Figs. 7 and 8 are compared with corresponding values calculated for a constant mass ratio of 27.41 which is the value for Mach number 0.80 from Fig. 6. Variations of flutter-speed index, flutter-frequency ratio, and reduced frequency with mass ratio are shown in Fig. 10 of Ref. 10.

Conclusion

Flutter calculations have been made by modified strip analysis for a supercritical wing model for which experimental flutter data and steady-state pressure distributions were previously available. Use of these pressure data to generate aerodynamic input for the flutter calculations produced a flutter boundary that is in excellent agreement with experiment in the subsonic range. In the transonic range, a "transonic bucket" was calculated which closely resembles the experimental one with regard to both shape and depth, but it occurs at about 0.04 Mach number lower than the experimental one. Some evidence indicates that this shift may be related to differences in model size relative to tunnel size for the pressure model and the flutter model, but this is not conclusive.

Nevertheless, the good results reported herein for the supercritical wing and the good results obtained previously for swept wings with conventional airfoils at subsonic, transonic, and supersonic speeds indicate that the modified strip analysis, in conjunction with aerodynamic parameters from steady-state experiments, is still useful for transonic speeds in the absence of validated nonlinear, three-dimensional, unsteady aerodynamic analysis methods.

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